

## Chapter 6 - Demand

As we saw in chapter 6, one thing we'll do with utility functions is to solve for the resulting demand functions

→ or sometimes we'll try to estimate a utility function from observed demands to do some welfare analysis

→ Demands will be functions of prices and incomes:

$$x_1 = x_1(p_1, p_2, m)$$

$$x_2 = x_2(p_1, p_2, m)$$

→ w/ these functions, we can do useful things like comparative statics

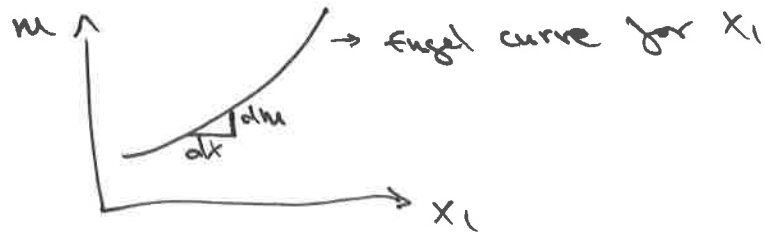
→ comparative statics show how an endogenous variable changes for a change in an exogenous variable or parameter

→ e.g.  $\frac{\partial x_1(p_1, p_2, m)}{\partial p_1}$  is a comparative static

→ these are useful b/c a retailer might care about how a change in price affects consumer demand.

### Engel Curves

→ an Engel curve shows how demand for a good changes as income changes:



→ the slope of the Engel curve is  $= \frac{\partial X(P_1, P_2, m)}{\partial m}$

→ we can find the Engel curve by tracing out the income-offer curve



→ this traps off all the demand for  $X_1$  at each value of  $m$

→ a normal good is a good for which demand increases as income increases:

$$\frac{\partial X}{\partial m} > 0$$

→ e.g. tropical vacations

→ normal goods have upward sloping Engel curves

→ inferior goods are goods whose demand falls as income rises:

$$\frac{\partial X}{\partial I} < 0$$

→ e.g. Ramen noodles

→ inferior goods have downward sloping Engel curves (at least once income is suitably high)

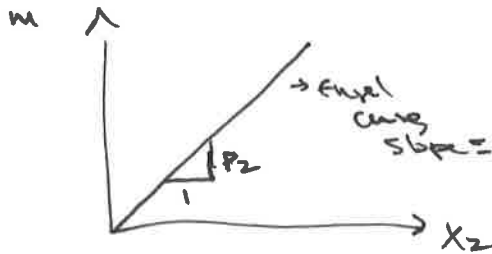
Examples:

Perfect subs.:  $u(x_1, x_2) = 2x_1 + x_2$

→ if  $p_1 > 2p_2$  then  $x_1^* = 0$   
 $x_2^* = \frac{M}{p_2}$

⇒ Engel curve =  $M(x_2) = p_2 x_2$

Just solve demand function for M



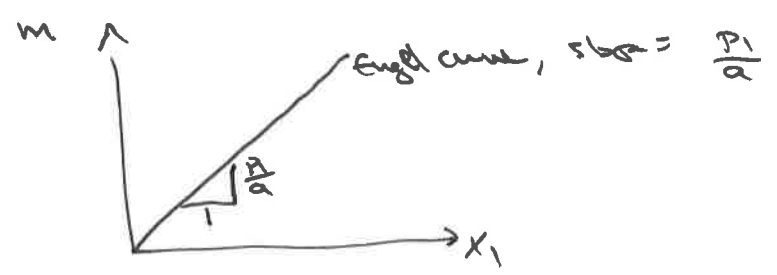
Cobb-Douglas:

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$\Rightarrow x_1^* = \frac{aM}{P_1}$$

$$x_2^* = \frac{(1-a)M}{P_2}$$

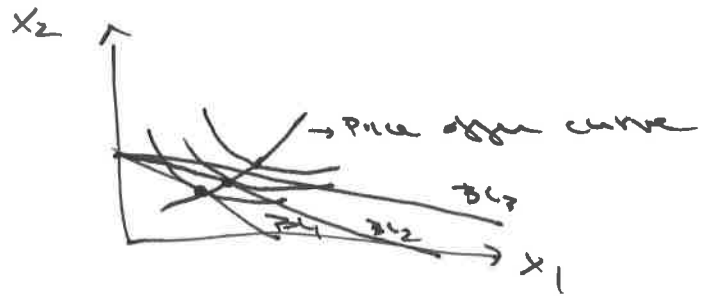
$\Rightarrow$  Engel curve for  $x_1 = \frac{P_1 x_1}{a}$



- $\rightarrow$  These 2 examples show homothetic preferences
  - $\rightarrow$  these are preferences where the consumer prefers goods in some ratio  $\rightarrow$  so as income increases, the amt. of goods consumed goes up, but the ratio is the same
  - $\rightarrow$  Homothetic preferences give rise to Engel curves that are straight lines
- $\rightarrow$  If Engel curves b/c steeper, the good is a necessary good
  - $\rightarrow$  its demand rises less quickly <sup>than</sup> income as income increases
- $\rightarrow$  If the Engel curve b/c becomes flatter, the good is a luxury good
  - $\rightarrow$  its demand rises more quickly than income as income increases

How demand changes as price change

→ We can plot and trace the optimal demands as price change:

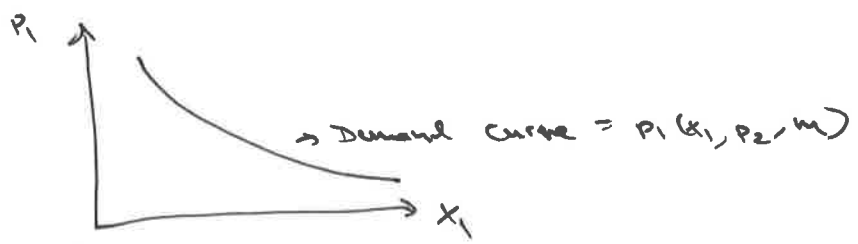


→ These gives us the price-offer curve

→ or we can just take the demand function and solve for price as a function of quantity demanded

$$x_1(p_1, p_2, M) \Rightarrow p_1(x_1, p_2, M)$$

→ this gives us the demand curve



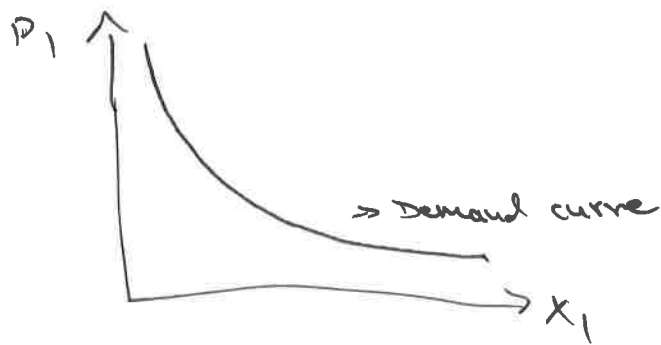
E.g Cobb-Douglas..

$$u(x_1, x_2) = x_1^a x_2^{1-a}$$

$$\Rightarrow x_1^* = \frac{aM}{p_1}$$

$$x_2^* = \frac{(1-a)M}{p_2}$$

$$\Rightarrow p_1 = \frac{aM}{x_1} = \text{demand curve}$$



→ we call a good a ordinary good if

$$\frac{\partial X_1}{\partial P_1} < 0$$

→ i.e. its demand falls as its price rises  
 → ordinary goods have downward sloping demand curves

→ we call a good a Giffen Good if

$$\frac{\partial X_1}{\partial P_1} > 0$$

→ i.e. demand increases as its price rises  
 (and demand falls as prices fall)  
 → Giffen goods have upward sloping demand curves

### Inverse Demand Functions

→ Note, when we solve demand for the demand curve, we cared about  $p(x_1)$  → or how does price change for diff quantities demanded

→ we held other prices and income constant when plotting the demand curve

→ The inverse demand function gives the price as a function of demand, prices, and incomes.

$$p_1(x_1, p_2, m)$$

→ really, the demand curve and the inverse demand function are the same

### Substitutes and Complements

→ we say that good 1 is a gross substitute for good 2 if:

$$\frac{\partial x_1}{\partial p_2} > 0$$

→ demand for good 1 increases as the price of good 2 increases

→ we say that good 1 is a gross complement to good 2 if:

$$\frac{\partial x_1}{\partial p_2} < 0$$

→ demand for good 1 decreases as the price of good 2 increases